**Tilt and Azimuth Angle for Optimal Peak Shaving Performance of Fixed-mount Photovoltaic Systems**

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**Abstract**

**Introduction**

Currently, photovoltaic (PV) systems provide less than one percent of the total energy consumed in the United States. To meet the total energy demand of the United States, approximately 10,000 square miles of solar panels would have to be constructed with the existing PV technology [1]. Although the technology to develop efficient PV systems with solar capacity is still under development, the use of PV systems as auxiliary energy sources have beneficial uses for consumers.

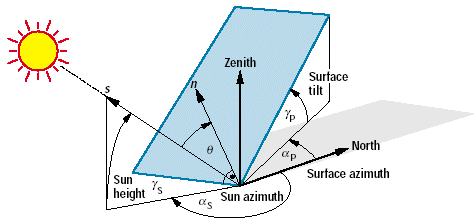
One such beneficial use of PV systems is the cost reduction of electrical energy for consumers during hours of high energy demand. The demand for electrical energy varies throughout a day. Typically, the peak demand hours are between 12 PM and 6 PM [2]. Electric utility companies charge consumers extra per kilowatt-hour during times of high energy demand. This extra charge is referred to as “peak use charge.”

Conveniently, irradiance, which is the density of solar radiation incident on a given surface, is close to the highest value (around 13: 00 averagely) during the peak demand hours. PV systems can take advantage of the “peak shaving” phenomenon, which describes how the electrical energy produced by a PV system can lower a consumer’s dependence on the electrical energy supplied by utility companies during peak demand hours and ultimately reducing the peak use charge. A slight change of tilt and azimuth angles of solar panels is needed to maximize the peak shaving effect.

Therefore, it is of great interest to determine the optimal positioning of the PV systems so that they illicit the greatest peak shaving performance. For fixed-mount PV systems, the PV array orientation is described by two angles:

1. *Panel tilt angle:* the angle between the array and the horizontal axis (usually the ground) and
2. *Panel azimuth angle (which is not to be confused with the solar azimuth angle):* the angle between the “projection of the normal line” of the panel surface and the north direction.

Figure 1 illustrates the various factors that affect a PV system’s ability to absorb solar energy and ultimately its ability to generate electric energy.



**Figure 1.** Illustration depicting the various angles associated with the orientation of a PV array [3]. Here, and are respectively the tilt angle and azimuth angle.

The objective of this research paper is to determine the optimal panel tilt and azimuth angle of fixed-mount PV systems that will result in the best peak shaving performance in Atlanta Georgia.

**Methodology**

In order to find the optimal panel azimuth and tilt angle for the PV systems to be implemented, a source of local hourly irradiance and solar azimuth angles for Atlanta was required. These data were obtained via the typical meteorological year data (TMY3) in 2009 [4].

Additionally, the interval of high energy demand must also be determined to ensure that the optimal panel azimuth and tilt angle were tailored to generate the best peak shaving performance for a realistic peak interval. The interval used in this investigation is from 12 PM to 6PM, according to the information from power providers.

The relationship between the energy output of a PV system and its orientation is given by the following equations:

Where:

= the angle of incidence of the sunlight on the panel

= the solar zenith angle

= the panel tilt angle

= the solar azimuth angle

= the panel azimuth angle

*I* = the insolation

= the direct normal insolation

In order to optimize the insolation, the panel azimuth and tilt angle must be values that bring (1) as close to one as possible. The panel tilt angle was determined first in order to evade the complications of having to deal with two unknown variables. Normally, under no peak shaving condition, the panel azimuth angle is set to180°, because Atlanta is in the northern hemisphere. However, in order to take peak shaving into consideration, the azimuth angle will be shifted to maximize the overall power supply from 12:00 pm to 6:00 pm, which is different from maximizing the 24 hours power supply consideration (where 180° is used). The angle and the zenith angle are obtained from the TMY3 data. The panel tilt angle and panel azimuth angle are found by using the MATLAB program in Appendix A.

After the panel tilt angle and azimuth angle were found, the optimal peak shaving net insolation is selected from 3195 samples, using the MATLAB program in Appendix A. Detailed explanations of the MATLAB program are provided that explain the process of finding the two angles and the net insolation for peak shaving purpose.

Next the comparison between peak-shaving PV system and non-peak-shaving system is made by calculating the net insolation of both systems during the peak demanding hours and the whole 24 hours separately. The algorithm for the complex calculations is shown in Appendix B.

The current PV systems are around 19 percent efficient [5]. Therefore to calculate the energy output of the PV system, the net insolation is multiplied by 0.19.

To evaluate the peak shaving performance, a mathematical model of the energy demand curve is needed. The energy demand curve shown in Graph 1 is modeled by a sinusoidal curve with a 24 hour period and a peak at 3 PM every day of the year. The ratio between the minimum and maximum point is two to one. This indicates that the energy demand at 3 PM is twice as large as the energy demand at 3 AM. Although this is a very simple mathematical model of the energy demand on a typical day in Atlanta, it is sufficient in presenting the peak shaving performances of a PV system.

The annual residential electrical energy consumption of Atlanta in 2005 was estimated to be around 23,377,516 MWh [6]. Therefore, the daily energy consumption of consumers is a little over 64,938 MWh. Since the energy demand curve is approximated by a cosine function and the peak energy demand hours is assumed to be from 12PM to 6PM, the percentage of the energy demand during the peak hours with respect to the total energy demand of the day can be calculated by taking the integral of the following function and dividing it by twice of the function evaluated from 0 to π:

3.

The peak demand energy is evaluated to be around 32.5 percent of the total energy demand per day. As a result, the energy demand during the hours of 12 PM to 6 PM in Atlanta per day is approximately 21106 MWh.

**Graph 1.** Energy demand curve with peak demand hours from 12 PM to 6 PM.

An assumption that needed to be made in order to assess the peak shaving performance of the PV system was the size of size of the PV system to be installed in Atlanta. It’s not economically feasible to install a PV system large enough to supply all of Atlanta’s electrical energy demands during the peak hours. Thus, it is much more realistic to assume that the PV system will provide for a percentage of the total electrical energy demand during the peak demand hours. Based upon the total contribution of solar energy in relation to the total energy generation in the United States, having the PV system provide for **one percent** of the total energy demand during the peak hour would be a good assumption.

**Results**

Using the data from TMY3 of 2009 for Atlanta, the MATLAB program takes in three columns and 9000 rows of the numerical data in TMY excel file with respect to each corresponding hour. Every hour’s 365 entries for the year are sampled out separately. Then the average is taken to calculate the average daily values across the year for solar azimuth angle, irradiance, and zenith angle.

After the three vector of values are obtained, the program extracts out the six peak hours’ data and goes through a large range of testing angles to find the optimal one that will maximize the term which can be seen as the coefficient of the term.

Then the program goes through another loop to find the best tilt angle that will maximize the result of equation 1, which is . Simply, the insolation can be determined then by multiplying the normal irradiance found earlier.

Similarly, the net insolation for 24 hours for both peak shaving and non-peak-shaving methods can be decided using the program.

Results Obtained:

For the purpose of peak shaving effect during the peak hours from 12:00 pm to 6:00 pm:

Optimal tilt angle = 36.1683°.

Optimal azimuth angle

Net average insolation for every peak hour = 359.284 W.

Use as the solar panel efficiency.

Electrical energy generated per day per square meter = 0.409583 Kwh.

Electrical power per square meter = 68.264 W.

For no peak shaving control group during the peak hours from 12:00 pm to 6:00 pm:

Optimal tilt angle = 32.5735°.

Optimal azimuth angle (fixed, according the reason introduced in methodology)

Net average insolation for every peak hour = 337.7 W.

Use as the solar panel efficiency.

Electrical energy generated per day per square meter = 0.385 Kwh.

Electrical power per square meter = 64.16 W. (94% of value get for best peak shaving effect).

For 24 hours energy production:

No peak shaving:

Optimal tilt angle = 31.8116°.

Optimal azimuth angle (fixed, according the reason introduced in methodology)

Net average insolation for every hour = 118.446 W.

Use as the solar panel efficiency.

Electrical energy generated per day per square meter = 0.54 Kwh.

Electrical power per square meter (average of 24 hours) = 22.5 W.

No peak shaving:

Net average insolation for every hour = 112.4 W.

Use as the solar panel efficiency.

Electrical energy generated per day per square meter = 0.5125 Kwh. (95% of no peak shaving)

Electrical power per square meter (average of 24 hours) = 21.353 W.

**Discussion**

In order to achieve the goal of supplying one percent power to Atlanta users during peak hours, a large PV station has to be build: (21106 MWh\*1%  **) /** (0.409583KWh) = 51,5300 square meters. The largest PV system in the world current occupies 2400 acres and this PV station design is achievable.

1KWh electricity is common charged with 5.72 cents during peak hours. With this consideration, the PV system can save roughly 4.41 million dollars for peak hours only. Although the cost of installing a PV station is still very high, new technological innovation may further reduce the cost of solar panels and at the same time increase its efficiency.

Furthermore, as global warming has tremendous negative influence on climate changes and biological diversity, immediate measures need to be taken to replace coal and natural gas power plants with clean energy resources like wind and solar energy. The Photovoltaic System will be put into larger use in the near future.

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**Appendix A**

function [betabest,val,val2,ind,suminso,totalinsolation] = pvPower(filename)

%Created by Tianhao Li. 07/21/2013. Georgia Institute of Technology.

% this program serves the purpose of processing the data from tmy 3 for

% Atlanta area. It also calculates the insolation for no reorientation, and

% the reoriented insolation in order to take account peak shaving effect.

% in the first part of the program, the code PROCESS ONLY the six hours

% peak electricity demand time, but in the second part the code decides the

% loss of total energy due to peak shaving effects.

[num] = xlsread(filename); %high level i/o to import the number values in the tmy3 file.

length = 8760; % the whole length of the rows of the values in the TMY3 data sheet.

sumvalue2=zeros(1,24); %initialized the values.

sumvalue3=zeros(1,24);

sumvalue4=zeros(1,24);

%this for loop gets the single hour solar irradiance, for example, for 12pm to 1pm, the average

%irradiance for the whole year is summation

%of the 365 days hour value and sum it

%separately for every hour in the 24 hours.

%additionally, the excel doesn't have easy

%function that can sum up the value

%separately from one whole column.

for time = 1:24

for i=time:24:length

sumvalue4(time) = sumvalue4(time) + num(i,4);%this is the summation of the fourth column in excel sheet.

end

end

sumvalue4 = sumvalue4./365; % take the daily average.

irradiance = sumvalue4; % solar irradiance for the

%the following part is not related to the changing of the azimuth angle.

for zenith = 1:24

for i=zenith:24:length

sumvalue2(zenith) = sumvalue2(zenith) + num(i,2); %apply same technique to acquire zenith angles.

end

end

sumvalue2 = sumvalue2./365;

zenith1 = sumvalue2;

%the end of the unchanged part, since we are not changing the zenith

%angle at all.

%cosine = cos(zenith1);

% sinevalue = sin(zenith1);

zenithpeak = zenith1(12:17);

cosinepeak = cos((zenithpeak./180)\*pi);

sinepeak = sin((zenithpeak./180)\*pi);

for azimuth = 1:24

for i=azimuth:24:length

sumvalue3(azimuth) = sumvalue3(azimuth) + num(i,3);%for azimuth angles.

end

end

% this is the end of exel information processing part.

sumvalue3 = sumvalue3./365; %daily average for azimuth angles.

azimu = sumvalue3;

azimupeak = azimu(12:17); %only peak hours concerned in the first part.

i=1; %initialize the index indicator.

for AzimuthChange = 180:1:250 %this for loop ranges from 180 to 250 in order to find the best azimuth angle for shaving effect.

%the reason of setting 180 to 250 is because 180 is the best for no

%shaving, around 1 pm, but shaving we concerned about 3pm, which is

%around 227, 250 goes to the last peak hour, so 250 is large enough

%to include the optimal choice.

AzimuthChange1 = AzimuthChange.\*ones(1,6); %change the size of the angle vector to match with 6 hours original solar angles.

peakvalue(i,:) = irradiance(12:17).\*cos(((AzimuthChange1-azimupeak)./180)\*pi).\*sinepeak; %calculate the socond coefficient for cos(beta).

%the above expression is simply cos(r-rs)\*sin(zenith), which is

%regared as the coefficient for cos(beta), beta is tilted angle we

%want to find, and r is the azimuth angle of the panels we want to

%find.

i= i+1; % so the final value for index i will be 72 for sure. The size of the peakvalue is 72\* 6.

end

secondfactor = peakvalue ; % this part changes. because of peakvalue is related to azimuth.

firstfactor = zeros(71,6); %initialize first vector, with size match up with 71 azimuth testing and 6 hours peak values.

for index = 1:71

firstfactor(index,:) = irradiance(12:17).\* cosinepeak; %firstfactor is simply cos(beta)\*irradiance.

%we want to include the irradiance in the term in order to get the highest insolation.

end

var1 = atan(secondfactor./firstfactor); %using the formula to sum up A\*sin(beta)+B\*cos(beta), which gives sqrt(A^2+B^2) \* cos(beta-alpha);

alpha = (var1.\*180)./pi;

R = sqrt(secondfactor.^2 + firstfactor.^2); %trying to find the biggest R with shifting azimuth angle.

beta = alpha(:,1)-1; %trials for beta starts at one unit value below the lowest alpha value of the six. But here, since we have 71 trials

%to test the azimuth, which in turn gives the

%alpha, so our alpha have 71\*6 values. And the

%first column has the lowest alpha values.

beta3 = beta;

finalvalue = zeros(71,45);

for i=1:45 % try 45 times due the range situation of the alpha angles.

beta = beta +1;

beta5 = (beta\*pi)/180;

beta2=[beta5 beta5 beta5 beta5 beta5 beta5]; %here beta

k = cos(beta2 - var1);

valsum = R.\*cos(beta2-var1); %get six hours insolation here.

k2 = valsum(:,1)+valsum(:,2)+valsum(:,3)+valsum(:,4)+valsum(:,5)+valsum(:,6); %one day insolation amount.

finalvalue(:,i) = k2;

end

[val2,ind] = max(finalvalue); % 45 maximum values for insolation every column indicating one sample of beta angle.

%45 indices for 45 degrees range of beta angles.

[val,ind2] = max(val2); % the largest insolation we can get for this peak shaving effect purpose from 12pm to 6pm.

ind = ind(ind2); %get the index for the array of azimuth angles and get the best azimuth angle, at the same time using the

%previous index value we get the best beta angle.

betabest = beta3(ind); %the best tilt angle for maximizing insolation.

val2 = finalvalue(1,:); %

[val2,indt] = max(val2); % the val2 value is the one without reorientation for peak shavings.

**Appendix B**

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%This is the start of the second part.

YOU NEED TO COMBINE THIS CODE WITH THE CODE IN APPENDIX A IN ORDER TO RUN IT CORRECTLY.

%FIRST WE CONSIDER THE TOTAL INSOLATION AND THE CORRESPONDING TILTED

%ANGLE.

zenithpeakT = zenith1(6:20); %everything remains same except that this time we take 24 hours into account.

azimupeakT = azimu(6:20); %notice that, don't consider 1 to 5 and 21 to 24 is because no insolation or related activities

irradT = irradiance(6:20); %happens at all, so value for that are zero or unuseful.

cosinepeakT = cos((zenithpeakT./180)\*pi);

sinepeakT = sin((zenithpeakT./180)\*pi);

peakvalueT = irradT.\*cos(((180-azimupeakT)./180)\*pi).\*sinepeakT;

secondfactor2 = peakvalueT ; % this part changes. Because of peakvalue is related to azimuth.

firstfactor2 = irradT.\* cosinepeakT;

var2 = atan(secondfactor2./firstfactor2); % find the alpha angle in the combined expression R\*cos(beta-alpha);

alpha2 = (var2.\*180)./pi;

R1 = sqrt(secondfactor2.^2 + firstfactor2.^2); %The amplitude for the new cos equation.

betat= min(alpha2)-1;

beta6 = zeros(1,15);

for i=1:177 %range changed to the 24 hours diverse alpha angles.

betat = betat +1;

betat2 = (betat\*pi)/180;

beta6(1,:)= betat2;

k = cos(beta6 - var2);

valsum2(i) = sum(R1.\*cos(beta6-var2));

end

[ totalinsolation indexb] = max(valsum2); % total insolation without changes for shaving effects.

%SECONDLY, SIMPLY CALCULATE THE TOTAL INSOLATION FOR SHAVING EFFECTS, WHICH

%MEANS THE INSOLATION FOR REORIENTED PV SYSTEMS FOR 24 RATHER 6 HOURS.

newbeta = 36.1683; %using the fixed optimal tilted angle calculated from part1.

newbeta = (newbeta\*pi)/180;

newazimuth = 214; %the optimal azimuth angle for shaving effect.

peakvaluek = irradT.\*cos(((newazimuth-azimupeakT)./180)\*pi).\*sinepeakT;

secondfactor3 = peakvaluek; %

var3 = atan(secondfactor3./firstfactor2); % find the alpha angle in the cobmbined experession R\*cos(beta-alpha);

R3 = sqrt(secondfactor3.^2 + firstfactor2.^2) ; %The amplitude for the new cos equation.

newbeta2 = zeros(1,15);

newbeta2(1,:) = newbeta;

newinsolation = R3.\*cos(newbeta2-var3);

suminso = sum(newinsolation); %total insolation per day for shaving-shifted solar panels.